Int. J. Heat Mass Transfer. Vol. 22, pp. 639–646
0017–9310/79/0501-0639 $02.00/0

FREE CONVECTION BETWEEN HORIZONTAL CONCENTRIC CYLINDERS IN A SLIGHTLY-THERMALLY STRATIFIED FLUID

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(*Received* 15 *May* 1978 *and in revised form* 18 *October* 1978)

Abstract—The present paper considers the steady, two-dimensional flow of a vertically stratified viscous fluid in the annulus between two concentric horizontal cylinders in a uniform gravity field. The outer cylinder is assumed to be maintained at a variable temperature such that conditions for vertical stratification are satisfied. Theoretical solutions are obtained in a power series of (modified) Grashof number G up to G^3. Two cases are considered: when the inner cylinder is either thermally insulated or when its surface temperature is kept constant. Results are presented mostly in the form of graphs of the streamlines and isotherms. A dimensionless stratification parameter S governs the flow. For S equal to zero, the solutions tend to the unstratified case. When S approaches infinity, the flow has both vertical and horizontal symmetry. When the inner cylinder is thermally insulated, the streamline pattern is almost the same as in the isothermal case ($S = \infty$), but the directions of the flow are reversed.

NOMENCLATURE

All primed quantities are dimensional; all unprimed quantities are dimensionless. Subscripted terms with m denote their corresponding values at the diametral plane—($y' = 0$).

- a', A', radii of the inner and the outer cylinders;
- g', acceleration of gravity;
- G, square root of modified Grashof number, $[g'\beta' a'^4 (dT'_\infty/dy')]^{1/2}/v'$;
- K', thermal conductivity;
- r', r, radial coordinate $r = r'/a'$;
- P, Prandtl number v'/α';
- R, ratio of the outer to the inner radii of the cylinders;
- S, steepness parameter defined as $[2a'(dT'_\infty/dy')]$ divided by the temperature difference between inner cylinder and the fluid occupying the diametral plane;
- T', T, temperature $T' = T'_m + a'(dT'_\infty/dy')]T$;
- dT_∞/dy, constant temperature gradient describing the constant stratification;
- v'_r, v_r, r—component of velocity, $v_r = v'_r a'/v' G$;
- v'_θ, v_θ, θ—component of velocity, $v_\theta = v'_\theta a'/v' G'$;
- y', y, $y' = r'\cos\theta$, $y = y'/a' = r'\cos\theta/a'$.

Greek symbols

- α', thermal diffusivity;
- β', volumetric coefficient of thermal expansion;
- θ, cylindrical polar angle measured from the upward vertical $\theta = 0$;
- ρ', density;
- v', kinematic viscosity;
- ψ', ψ, stream function $\psi = \psi'/Gv'$.

1. INTRODUCTION

NATURAL convection heat transfer within enclosed spaces is becoming increasingly important because of its various applications in nuclear reactor design, cooling of electronic equipment, aircraft cabin insulation, pressurized-gas, underground electric transmission cables and thermal storage systems. Experimental, analytical and numerical studies of free convection problems in the annulus between horizontal concentric cylinders have been presented by several authors for small and large Grashof numbers [1–11].* Thermoconvective motion of low Prandtl number fluids within a horizontal cylindrical annulus has been investigated by Custer and Shaughnessy [17] using a double perturbation expansion in powers of the Grashof and Prandtl numbers. Eichhorn, Lienhard and Chen [12] gave experimental heat transfer results for isothermal spheres and horizontal cylinders immersed in a thermally stratified fluid. They also presented visual observations of the flow field for the sphere for various values of the steepness parameter S. Observations on convective transport and plume shedding induced by a heated horizontal cylinder submerged in quiescent salt-stratified water have been made by Hubbell and Gebhart [13]. Chen [14] in his thesis studied the free convection from a horizontal cylinder in stratified fluid both analytically and experimentally (see also Chen and Eichhorn [15]).

In this paper, the free convection problem between two concentric horizontal cylinders is presented when the (modified) Grashof number G is small. The

*An extensive literature survey has been given by Kuehn and Goldstein [11].

fluid inside the annulus is considered thermally stratified. It is obtained by imposing a vertical temperature gradient on the walls of the outer cylinder. The inner cylinder is either maintained at a constant temperature or is thermally insulated. Perturbation solutions in powers of G are obtained up to G^3. Streamlines and isotherms are plotted for various values of S for a radius ratio $R = 2$, $P = 0.7$ and $G = 1$. The velocity components and the Nusselt numbers are also shown graphically for the above mentioned values of the parameters.

2. FORMULATION AND SERIES SOLUTION

Consider two infinite concentric circular cylinders whose common axis is horizontal. The region inside the annulus contains a viscous incompressible fluid and a uniform gravity field acts vertically downward. Cylindrical coordinates are used, the angular coordinate being measured clockwise from the upward vertical $\theta = 0$. The Navier–Stokes equations for steady, two-dimensional motion are (see Mack and Bishop [5]):

$$\nabla^4\psi = -\frac{G}{r}\frac{\partial(\psi, \nabla^2\psi)}{\partial(r, \theta)} - G\left(\sin\theta\frac{\partial T}{\partial r} + \frac{\cos\theta}{r}\frac{\partial T}{\partial\theta}\right), \quad (1)$$

$$\nabla^2 T = \frac{GP}{r}\frac{\partial(T, \psi)}{\partial(r, \theta)}, \quad (2)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

The above equations make use of the Boussinesq approximation (see Spiegal and Veronis [16]).

$$\frac{\rho' - \rho'_m}{\rho'_m} = -\beta'(T' - T'_m), \qquad T' = T'_m + a'(\mathrm{d}T'_x/\mathrm{d}y)T. \quad (3)$$

The non-dimensional velocity components are related to ψ by

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad v_\theta = -\frac{\partial\psi}{\partial r}. \quad (4)$$

The boundary conditions are

$$\psi = \frac{\partial\psi}{\partial r} = 0, \; T = \frac{1}{S} \text{ or } \frac{\partial T}{\partial r} = 0 \text{ at } r = 1, \quad (5)$$

$$\psi = \frac{\partial\psi}{\partial r} = 0, \quad T = r\cos\theta \text{ at } r = R. \quad (6)$$

Three parameters G, P and S govern the flow. We attempt to solve (1) and (2) subject to conditions (5) and (6) by an expansion in powers of G (P and S fixed). For the solution, we assume the form,

$$\psi = \psi_0(r, \theta) + G\psi_1(r, \theta) + G^2\psi_2(r, \theta) + \ldots, \quad (7)$$

$$T = T_0(r, \theta) + GT_1(r, \theta) + G^2T_2(r, \theta) + \ldots \quad (8)$$

and substitute into equations (1) and (2). Since ψ and $\partial\psi/\partial r$ vanish both at $r = 1$ and $r = R$, ψ_0 is zero throughout. By similar arguments T_1, T_3, $T_5 \ldots$ and ψ_2, $\psi_4 \ldots$ etc. are zero. Equation (2) gives for T_0

$$\nabla^2 T_0 = 0. \quad (9)$$

Solutions of (9) satisfying (5) and (6) are

$$T_0 = \frac{1}{S}\left(1 - \frac{\ln r}{\ln R}\right) + \frac{R^2}{R^2 - 1}(\cos\theta)\left(r - \frac{1}{r}\right) \quad (10)$$

or

$$T_0 = \frac{R^2}{R^2 + 1}(\cos\theta)\left(r + \frac{1}{r}\right). \quad (11)$$

Equations (10) and (11) represent the solution for the isothermal and thermally-insulated inner cylinder cases respectively. Substitution of (10) and (11) into (1) gives

$$\nabla^4\psi_1 = \frac{1}{S\ln R}\frac{\sin\theta}{r} - \frac{R^2(\sin 2\theta)}{(R^2 - 1)r^2} \quad (12)$$

and

$$\nabla^4\psi_1 = \frac{R^2}{R^2 + 1}\frac{\sin 2\theta}{r^2} \quad (13)$$

whose solutions, satisfying the conditions $\psi_1 = \partial\psi_1/\partial r = 0$ both at $r = 1$ and $r = R$ are

$$\psi_1 = \frac{\sin\theta}{S(16\ln R)}\left(B_1r^3 + r^3\ln r + B_2r + B_4r\ln r + \frac{B_3}{r}\right) + \frac{\delta R^2(\sin 2\theta)}{16(R^2 + \delta)}\left(C_1r^4 + C_2r^2 - r^2\ln r + C_3 + \frac{C_4}{r^2}\right) \quad (14)$$

and

$$\psi_1 = \frac{\delta R^2(\sin 2\theta)}{16(R^2 + \delta)}\left(C_1r^4 + C_2r^2 - r^2\ln r + C_3 + \frac{C_4}{r^2}\right), \quad (15)$$

where

$$B_0 = -4(R^2 - 1)[(R^2 - 1) - (R^2 + 1)\ln R],$$
$$B_1 = [(R^2 - 1)^2 + 2(R^2 - 1)^2\ln R - 4R^4\ln^2 R]/B_0,$$
$$B_2 = [(R^2 - 1)(1 - R^4) - 2(R^2 - 1)^2\ln R + 8R^4\ln^2 R]/B_0,$$
$$B_3 = 2(R^2 - 1)(R^4 - 1 - 4R^2\ln R)/B_0,$$
$$B_4 = R^2[(R^2 - 1)^2 - 4R^2\ln^2 R]/B_0$$

and

$$C_1 = \frac{-(R^2 + 1)}{2(R^2 - 1)^2} - \frac{2R^2\ln R}{(R^2 - 1)^3},$$
$$C_2 = \frac{-(R^4 + 4R^2 + 1)}{2(R^2 - 1)^2} + \frac{R^2(R^4 + R^2 + 4)}{(R^2 - 1)^3}\ln R;$$
$$C_3 = -(3C_1 + 2C_2 - 1/2),$$
$$C_4 = 2C_1 + C_2 - 1/2$$

and δ is equal to -1 in (14) and $+1$ in (15) respectively.

Similar expressions for T_2 and ψ_3 can be obtained using (1), (2), (10), (11), (14) and (15) as given by the following:

$$T_2(r, \theta) = \sum_{n=1}^{3} f_n(r)(\cos n\theta),$$

$$\Psi_3(r, \theta) = \sum_{m=1}^{4} F_m(r)(\sin m\theta).$$

The coefficients f_n and F_m are functions of r, R, P and S. These expressions are long and are omitted to conserve space; readers interested in them are invited to write to the authors.

by the dominant term ψ_1, and the radial velocity component $v_r(\delta = -1)$ given by the following:

$$\psi_1 = \frac{\delta R^2 \sin 2\theta}{16(R^2+\delta)} \times \left[C_1 r^4 + C_2 r^2 - r^2 \ln r + C_3 + \frac{C_4}{r^2} \right] \quad (16)$$

and

$$V_r = \frac{1}{r}\frac{\partial \psi_1}{\partial \theta} = \frac{\delta R^2 \cos 2\theta}{8(R^2+\delta)} \times \left[C_1 r^4 + C_2 r^2 - r^2 \ln r + C_3 + \frac{C_4}{r^2} \right]. \quad (17)$$

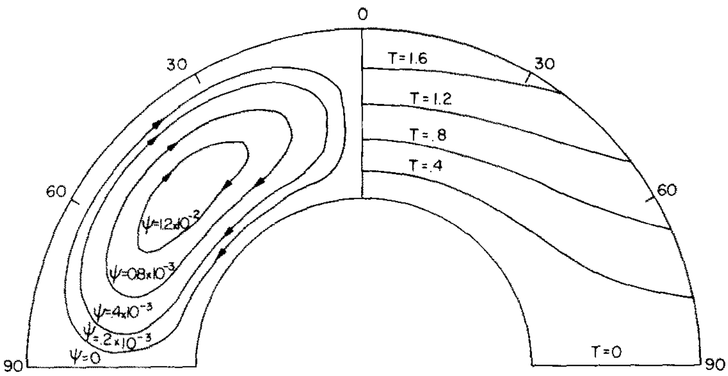

FIG. 1. Streamlines and isotherms for $S = \infty$, $G = 1$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 45$ and $135°$.

3. DISCUSSION

In the preceding section, the two-term expansions for T and ψ have been obtained in powers of G, with S, P and R fixed. According to the criterion discussed by Mack and Bishop [5] and Hodnett [18], we find that the series solution converges for $G \leq 10$ when $R = 2$ and P and $S \simeq 0(1)$. When $R = 5$, the solution is valid for $G \leq 2$, P and S being of order unity.

Behavior of the streamlines and isotherms is presented in detail for fixed values of $R(=2)$, $G(=1)$ and $P(=0.7)$ for various values of S ranging from zero to infinity. For the steepness parameter S tending to zero, the limit of the solution obtained in this paper approaches the unstratified case of Mack and Bishop [5]. Streamlines for this case when the inner cylinder is maintained at a higher temperature than that of the outer cylinder have been shown graphically by several authors [2, 5, 11]. It is found that the flow is symmetrical with respect to the vertical plane ($\theta = 0$ and π) through the axis of the cylinders. The flow is upward along the inner cylinder and downward along the outer cylinder. The motion in the right half annulus is clockwise near the outer cylinder and counter-clockwise at the inner. But when S tends to infinity, the flow becomes symmetrical with respect to both the vertical and horizontal planes ($\theta = \pi/2$ and $3\pi/2$) passing through the axis of the cylinders. This can be shown v_r is negative at $\theta = 0$, vanishes at $\theta = \pi/4$ and becomes positive at $\theta = \pi/2$, near the inner cylinder. An inflow at $\theta = 0$ from the outer cylinder to the inner cylinder changes into an outflow at $\theta = \pi/2$, transition taking place at $\theta = \pi/4$. Near the outer cylinder an exactly reverse flow takes place. A similar streamline picture was observed by Eichhorn *et al.* [12] in their flow visualization for a sphere in a stratified medium. The streamlines for this stratified case are shown in Fig. 1.

When the streamlines are sketched for various finite values of S greater than zero, the interaction of the two free convection flows ($S = 0$ and $S = \infty$) is described as follows. For smaller values of S, a single cell of the crescent-eddy type of flow (unstratified case) exists in the two halves of the annulus. But as S is increased (perhaps to a value of 0.8), the single cell flow changes into a double cell flow, and a region of reversed flow exists near $\theta = 0$. A further increase in the value of S moves the angle of separation of one cell from the other toward $\theta = \pi/2$.

Streamlines for fixed values of $G = 1.0$, $P = 0.7$ and $S = 1$, 3.33 and 10 are shown graphically in Figs. 2(a), (b) and (c) respectively. These flow lines depict the motion discussed above. The velocity components v_r and v_θ are plotted vs radial position for various values of θ in Figs. 3 and 4. v_r vanishes and changes sign at $\theta = 20$ and 120 for $S = 1$, at $\theta = 40$

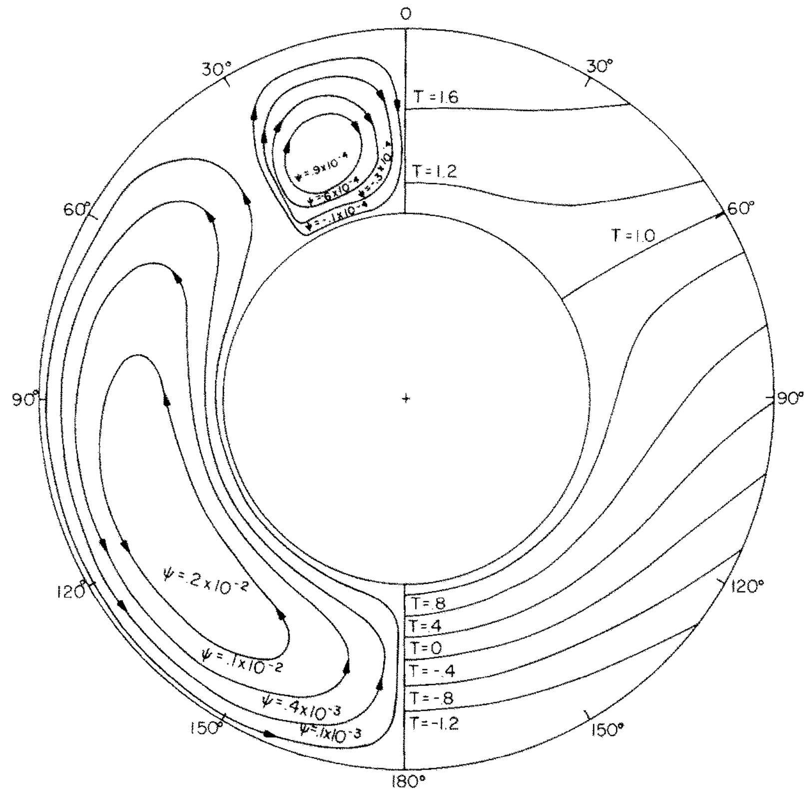

FIG. 2(a). Streamlines and isotherms for $S = 1$, $G = 1$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 20$ and $120°$.

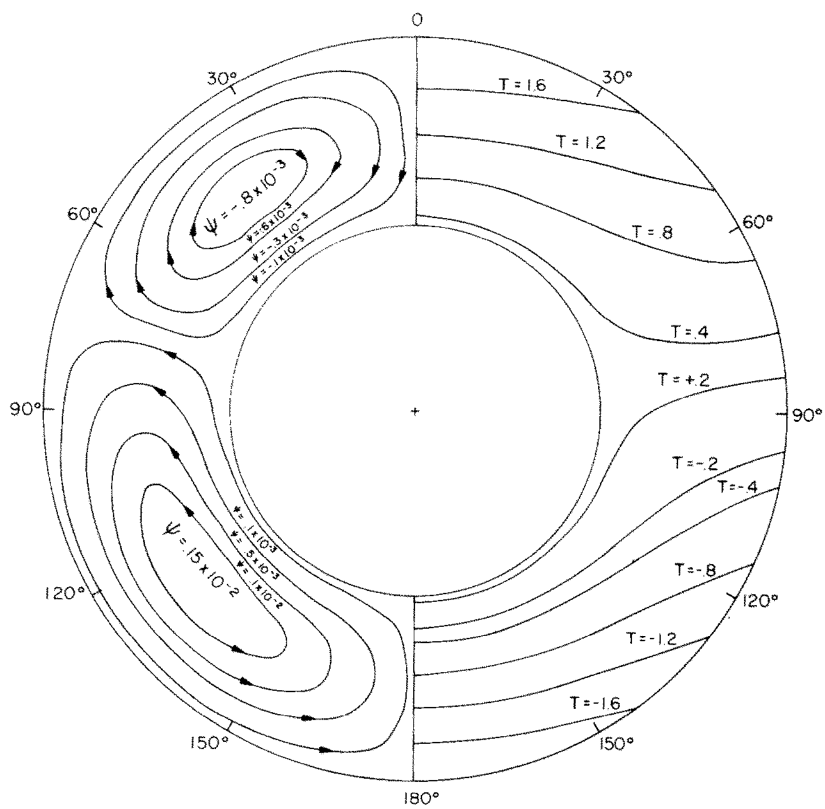

FIG. 2(b). Streamlines and isotherms for $S = 3.33$, $G = 1$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 40$ and $130°$.

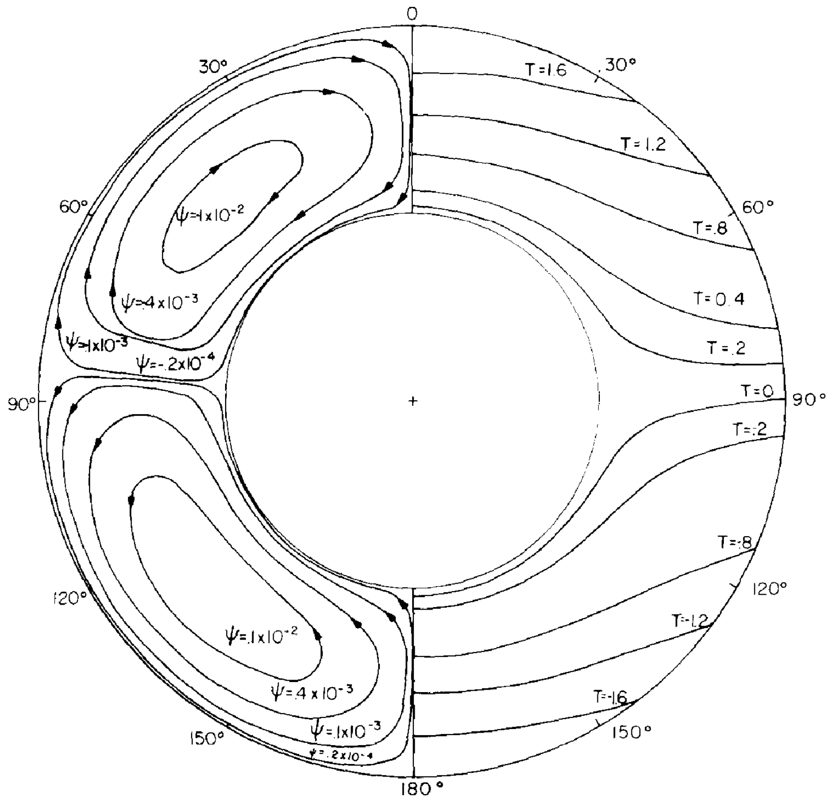

FIG. 2(c). Streamlines and isotherms for $S = 10$, $G = 1$, $P = 0.7$, $R = 2$. Radial velocity changes sign at $\theta = 45$ and $135°$.

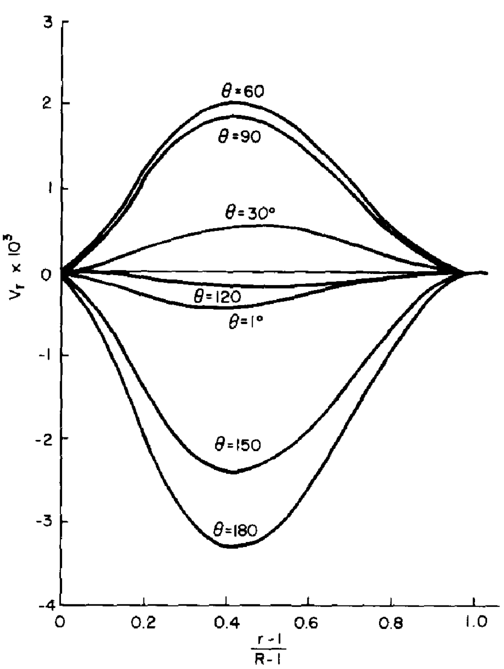

FIG. 3(a) Radial component of velocity vs radial position for $S = 1$, $G = 1$, $P = 0.7$, $R = 2$. v_r vanishes and changes sign at $\theta = 20$ and $120°$.

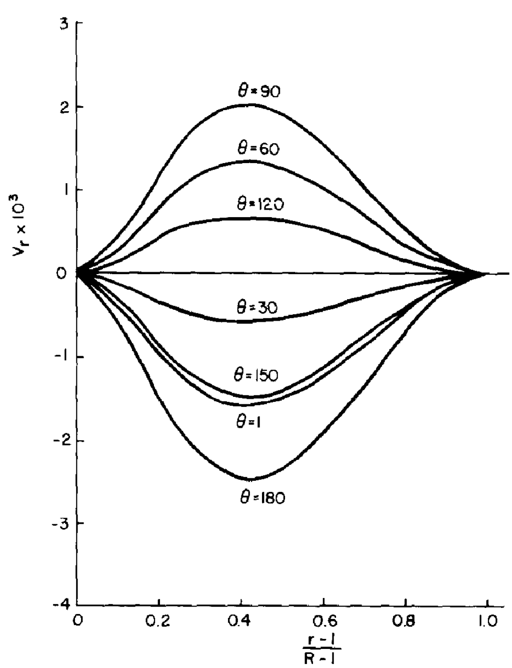

FIG. 3(b). Radial component of velocity vs radial position for $S = 3.33$, $G = 1$, $P = 0.7$, $R = 2$. v_r vanishes and changes sign at $\theta = 40$ and $130°$.

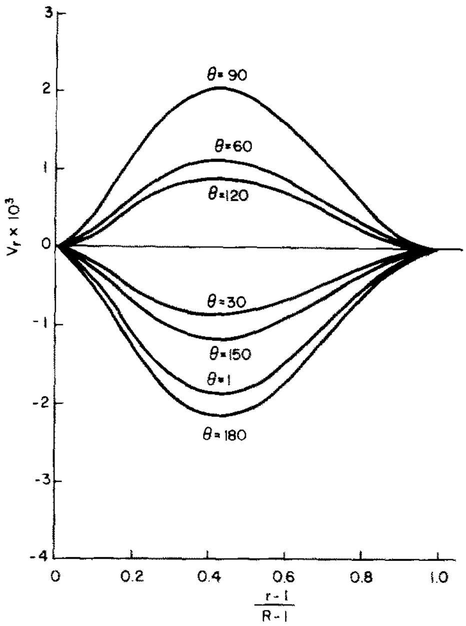

FIG. 3(c). Radial component of velocity vs radial position for $S = 10$, $G = 1$, $P = 0.7$, $R = 2$. v_r vanishes and changes sign at $\theta = 45$ and 135°.

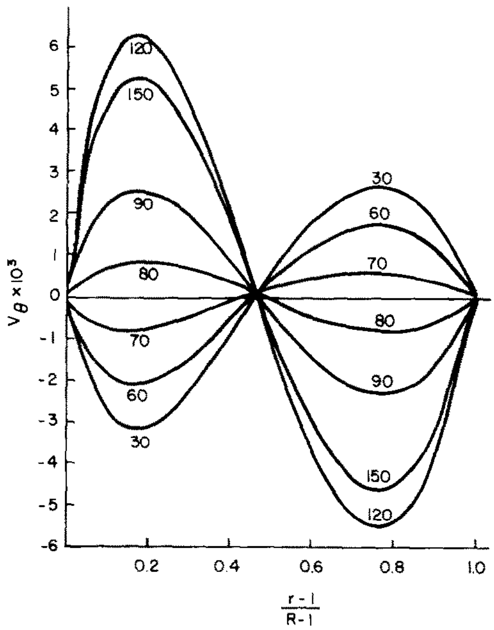

FIG. 4(b). Theta component of velocity vs radial position for $S = 3.33$, $G = 1$, $P = 0.7$, $R = 2$. v vanishes and changes sign at $\theta = 75°$.

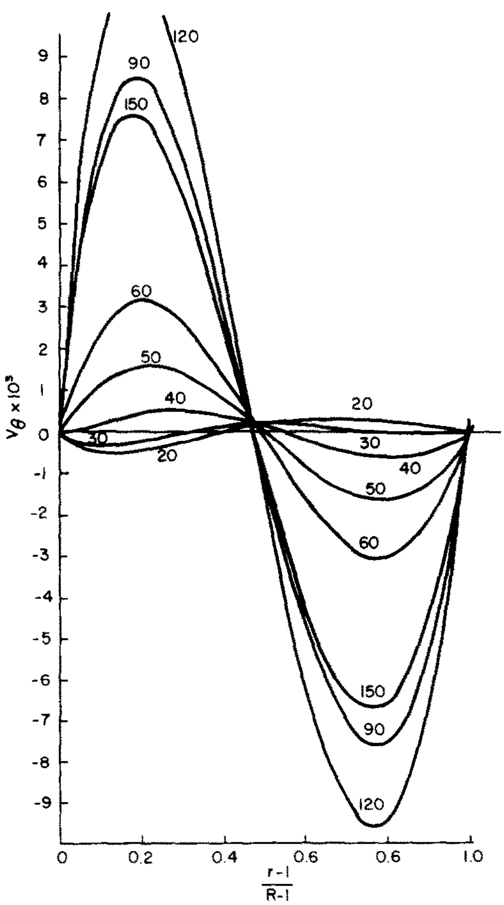

FIG. 4(a). Theta component of velocity vs radial position for $S = 1$, $G = 1$, $P = 0.7$, $R = 2$. v_θ vanishes and changes sign at $\theta = 35°$.

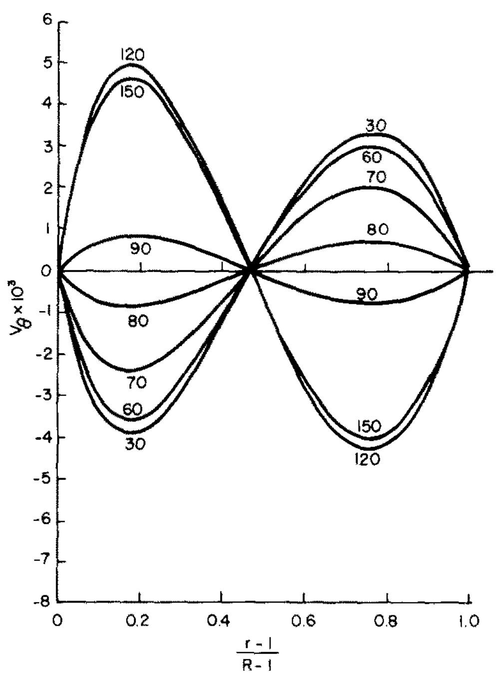

FIG. 4(c). Theta component of velocity vs radial position for $S = 10$, $G = 1$, $P = 0.7$, $R = 2$. v_θ vanishes and changes sign at $\theta = 135°$.

and 130 for $S = 3.33$ and $\theta = 45$ and 135 for $S = 10$. v_θ vanishes and changes sign at $\theta = 35$ for $S = 1.0$, at $\theta = 75$ for $S = 3.33$ and at $\theta = 85$ for $S = 10$.

When the inner cylinder is thermally insulated one finds from the dominant terms $\psi_1(16)$, $v_r(17)$ and $\delta = +1$, that all the qualitative features of the flow will be the same as for $S = \infty$, but with the flow

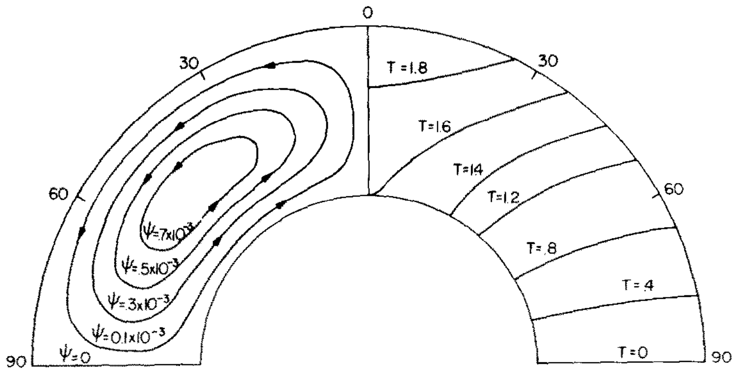

FIG. 5. Streamlines and isotherms for the case with the inner cylinder thermally insulated.

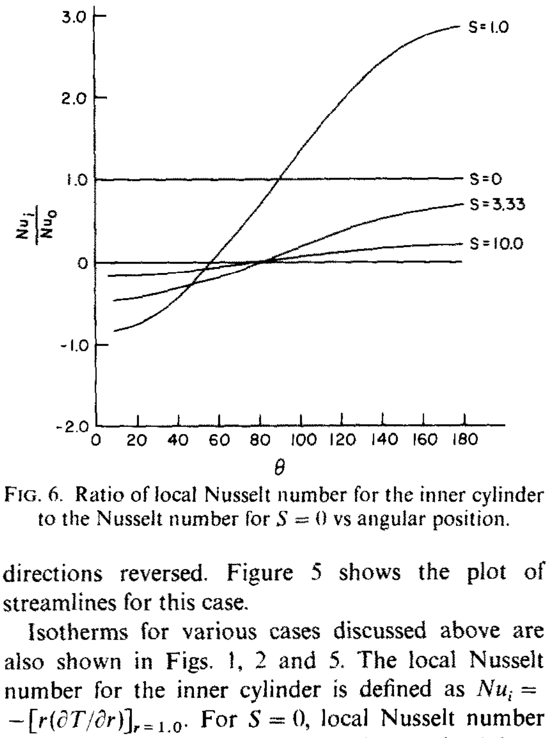

FIG. 6. Ratio of local Nusselt number for the inner cylinder to the Nusselt number for $S = 0$ vs angular position.

directions reversed. Figure 5 shows the plot of streamlines for this case.

Isotherms for various cases discussed above are also shown in Figs. 1, 2 and 5. The local Nusselt number for the inner cylinder is defined as $Nu_i = -[r(\partial T/\partial r)]_{r=1.0}$. For $S = 0$, local Nusselt number for the inner cylinder $Nu_{s=0}$ can be obtained from the calculations of Mack and Bishop [5]. We find that for $G = 1$, $Nu_{s=0}$ depends only on the first term of the expansion, considering up to the second decimal place.

$$Nu_{s=0} = 1/\ln R. \tag{18}$$

Figure 6 shows the plot of $(Nu_i/Nu_{s=0})$ vs θ for various values of S. It is found that the local Nusselt number for the inner cylinder decreases with S.

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CONVECTION NATURELLE ENTRE DES CYLINDRES CONCENTRIQUES ET HORIZONTAUX DANS UN FLUIDE LEGEREMENT STRATIFIE THERMIQUEMENT

Résumé—On considère l'écoulement permanent, bidimensionnel d'un fluide visqueux stratifié verticalement dans un espace annulaire entre deux cylindres concentriques et horizontaux, dans un champ uniforme de pesanteur. Le cylindre extérieur est supposé maintenu à une température variable de telle sorte que soient satisfaites les conditions de stratification verticale. On obtient des solutions théoriques en série de puissances du nombre de Grashof (modifié) G jusqu'à G^3. Deux cas sont considérés: le cylindre intérieur est isolé thermiquement, ou sa température de surface est constante. Des résultats sont présentés sous forme de graphes d'isothermes et de lignes de courant. Un paramètre adimensionnel de stratification S' gouverne l'écoulement. Pour S égal à zéro, les solutions tendent vers le cas non-stratifié. Quand S tend vers l'infini, l'écoulement a, à la fois, une symétrie verticale et une horizontale. Quand le cylindre interne est thermiquement isolé, la configuration des lignes de courant est à peu près la même que dans le cas isotherme ($S = \infty$), mais les directions de l'écoulement sont inversées.

FREIE KONVEKTION ZWISCHEN HORIZONTALEN KONZENTRISCHEN ZYLINDERN IN EINEM SCHWACH THERMISCH GESCHICHTETEN MEDIUM

Zusammenfassung—In der vorliegenden Arbeit wird die stationäre zweidimensionale Strömung einer vertikal geschichteten viskosen Flüssigkeit im Ringraum zwischen zwei konzentrischen horizontalen Zylindern unter der Annahme eines gleichförmigen Gravitationsfeldes betrachtet. Dabei wird angenommen, daß der äußere Zylinder auf einer variablen Temperatur gehalten wird, so daß die Bedingungen für vertikale Schichtung erfüllt sind. Theoretische Lösungen werden in Form einer Potenzreihe 3. Grades der (modifizierten) Grashof-Zahl erhalten. Dabei werden zwei Fälle untersucht: Der innere Zylinder wird entweder thermisch isoliert oder mit konstanter Oberflächentemperatur angenommen. Die Ergebnisse werden hauptsächlich durch grafische Darstellung der Stromlinien und Isothermen angegeben. Ein dimensionsloser Schichtungsparameter S bestimmt die Strömung. Für S gleich null geht die Lösung in den nichtgeschichteten Fall über. Geht S gegen unendlich, so ist der Fluß sowohl vertikal als auch horizontal symmetrisch. Ist der innere Zylinder thermisch isoliert, so haben die Stromlinien fast den gleichen Verlauf wie im isothermen Fall ($S = \infty$), aber die Strömungsrichtung ist umgekehrt.

СВОБОДНАЯ КОНВЕКЦИЯ МЕЖДУ ГОРИЗОНТАЛЬНЫМИ КОНЦЕНТРИЧЕСКИМИ ЦИЛИНДРАМИ В СЛАБО ТЕРМИЧЕСКИ СТРАТИФИЦИРОВАННОЙ ЖИДКОСТИ

Аннотация — Исследуется стационарное двумерное течение вертикально стратифицированной вязкой жидкости в кольцевом зазоре между двумя концентрическими горизонтальными цилиндрами в однородном гравитационном поле. Предполагается, что температура внешнего цилиндра изменяется, так что удовлетворяются условия для стратификации жидкости в вертикальном направлении. Получены теоретические решения в виде степенного ряда (модифицированного) числа Грасгофа G до значений G^3. Исследуются случаи, когда внутренний цилиндр или термически изолирован, или температура его поверхности поддерживается постоянной. Результаты представлены в основном в виде диаграмм линий тока и изотерм. Безразмерный параметр стратификации S является определяющим для потока. При $S = 0$ решения стремятся к случаю отсутствия стратификации. При $S = \infty$ поток характеризуется как вертикальной, так и горизонтальной симметрией. В случае, когда внутренний цилиндр является термически изолированным, картина линий тока почти совпадает с изотермическим случаем ($S = \infty$), но с обратным направлением потока.